PROBLEM LIST
CONFERENCE ON 4–MANIFOLDS AND KNOT CONCORDANCE

MPIM, OCTOBER 17-21, 2016

Problem 1 (Lukas Lewark)**.** Which Seifert matrices A come from negative amphicheiral knots? Note that A is S–equivalent to $-A^T$ and $\det(t^2A - A^T) \doteq f(t)f(t^{-1})$ and $f(t^{-1}) \doteq f(-t)$ by Hartley–Kawauchi. For example, we don't know whether $A = \begin{bmatrix} -15 & 3 \\ 2 & 5 \end{bmatrix}$ is a Seifert-matrix of a negative amphicheiral knot.

Problem 2 (David Gay)**.** Produce two trisections of the same 4–manifold X with the same genus and prove that they are not diffeomorphic. We say two trisections of X, (X_1, X_2, X_3) and (X_1', X_2', X_3') are *diffeomorphic* if there is a self-diffeomorphism $f\colon X \to X$ whose restrictions are diffeomorphisms $f|_{X_i}\colon X_i \to X_i'$. By Gay–Kirby, these become diffeomorphic after stabilizations. Peter Teichner's comment: Maybe, this is a 3–manifold question or a 3–complex question about the spine.

Problem 3 (Matt Hedden)**.** There are certain maps from the set of isotopy classes of knots $\mathcal{K}$ to the set of homeomorphism classes of 3-manifolds $\mathcal{M}$ (or to $\mathcal{K}$) which descend to (rational or $\mathbb{Z}_p$) homology cobordism or knot concordance. For example, consider p/q-surgeries $S^3_{p/q}(K)$, p^k-fold branched covers $\Sigma_{p^k}(K)$ for any prime p, satellite operations $P(K)$. Let Ψ_p be the set of homology cobordism classes of homology $L(p,1)$. Let $\Theta_p^{\mathbb{Z}_p}$ be the $\mathbb{Z}_p$-homology cobordism group of $\mathbb{Z}_p$-homology 3-spheres.

1. Is the map $\mathcal{C} \to \prod_{p,q\in\mathbb{Z}} \Psi_p$ given by $[K] \mapsto ([S^3_{p/q}(K)])_{p,q\in\mathbb{Z}}$ injective?
2. Is the map $\mathcal{C} \to \prod_{p:\text{prime},k\in\mathbb{N}} \Theta_p^{\mathbb{Z}_p}$ given by $[K] \mapsto ([\Sigma_{p^k}(K)])$ injective?

Problem 4 (Matt Hedden)**.** Is there a pattern P such that $P\colon \mathcal{C} \to \mathcal{C}$ is a nontrivial homomorphism?

Problem 5 (Adam Levine)**.** Suppose that a $\mathbb{Z}$–homology 3-sphere Y bounds a $\mathbb{Z}$–homology 4–ball. Can Y bound a contractible, smooth 4–manifold? Given $a \in \pi_1(Y)$ is there a $\mathbb{Z}$–homology 4–ball X such that a vanishes in $\pi_1(X)$? (If Y bounds a contractible 4–manifold, the latter question would work.)

Problem 6 (Danny Ruberman)**.** Recall that a surgery exact sequence for a 4-dimensional Poincaré complex X (with $\pi_1(X)$ *good*) is

$$L_5(\mathbb{Z}[\pi_1(X)]) \to \mathcal{S}^{\text{TOP}}(X) \to \mathcal{N}^{\text{TOP}}(X) \to L_4(\mathbb{Z}[\pi_1(X)]).$$

Here $\mathcal{N}(X)$ is the abelian group of normal invariants, and the map from $L_5(\mathbb{Z}[\pi_1(X)]$ is really an action of the L-group on the topological structure set. In the smooth case, this sequence is known to not be exact. On the other hand, the normal map $\mathcal{S}(X) \to \mathcal{N}(X)$ can be non-trivial, as for example the work of Cappell-Shaneson and Fintushel-Stern shows for $X = RP^4$. A broad question is whether one can realize the action of L_5 on the *smooth* structure set, to get an exotic smooth manifold (rather than an exotic homotopy equivalence). A good specific instance arises when $\pi = Z \times Z_n$ with $n \geq 3$. Freedman's theorem gives, via the action of L_5, many topological 4–manifolds that are homotopy equivalent to $S^1 \times L(p,q)$ but not homeomorphic to $S^1 \times L(p,q)$. Are the manifolds you get *in this manner* smoothable?

Problem 7 (Daniele Celoria)**.** Is there a winding number one pattern P such that $P(U)$ is slice and $g_4(P(K)) < g_4(K)$ for some knot K? If so, does P lower slice genus for all knots? or for just one knot?

Problem 8 (Sebastian Baader)**.** Is $|s(K)+\frac{\sigma(K)}{2}| \leq \mathrm{width}_{Kh} - 2$ for all knots (or thin knots) K?

Problem 9 (Adam Levine)**.** Cochran–Franklin–Hedden–Horn gave non-concordant knots whose 0–surgery manifolds are homology cobordant preserving the homology class of meridians (that is, meridians are homologous in the homology cobordism). Yasui gave non-concordant knots whose 0–surgery manifolds are homeomorphic. Suppose that $S^3_0(K_1)$ and $S^3(K_2)$ are homology cobordant (or homeomorphic) preserving (free) homotopy classes of meridians. Are K_1 and K_2 concordant? Do annulus twists preserve meridians?

Problem 10 (Matt Hedden)**.** Is there an algorithm which determines whether a given knot is ribbon or not (e.g. using some analogue of normal surface theory)?

Problem 11 (Matt Hedden)**.** Let $\mathcal{L}^{top}_{\mathbb{Q}}$ and $\mathcal{L}_{\mathbb{Q}}$ be the subgroups of the topological rational homology cobordism group $\Theta^{\mathrm{top}}_{\mathbb{Q}}$ and the smooth rational homology cobordism group $\Theta_{\mathbb{Q}}$ generated by lens spaces, respectively. Lisca proved $\mathcal{L}_{\mathbb{Q}} \cong \mathbb{Z}^\infty \oplus \mathbb{Z}_2^\infty$. Is the natural map $\mathcal{L}^{top}_{\mathbb{Q}} \to \mathcal{L}_{\mathbb{Q}}$ an isomorphism?

Problem 12 (Tetsuya Abe)**.** For a knot K, let $X_0(K)$ be the trace of 0–surgery on a knot K, that is $X_0(K) = B_4 \cup_K$ (2-handle) where the 2–handle is attached along the zero framing of K. Suppose that $X_0(K)$ is diffeomorphic to $X_0(K')$. Is $s(K) = s(K')$ or $\tau(K) = \tau(K')$?

Problem 13 (Jeff Meier)**.** Given a braid $\beta \in B_{2b}$, characterize when the diagram in Figure 1 satisfies that $\mathcal{P}_{ij} \cup \overline{\mathcal{P}_{jk}}$ is an unlink for all $\{i,j,k\} = \{1,2,3\}$. This will give a bridge-trisection of a knotted surface in S^4.

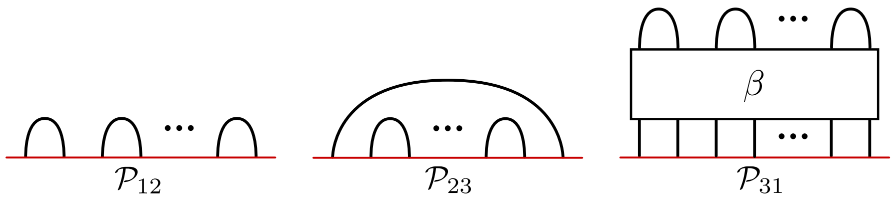

FIGURE 1. A tri-plane diagram.

Problem 14 (Steven Sivek)**.** Let K be a knot and $\Lambda \subset (S^3, \xi_{std})$ be a Legendrian knot whose smooth knot type is K. The slice–Bennequin inequality says $tb(\Lambda) \leq 2g_4(K) - 1$. If $\Lambda = \partial L$ where L is a Lagrangian surface in B^4, then $tb(\Lambda) = 2g_4(K) - 1$, in fact, this equals $2g(L) - 1$. That is, a Lagrangian surface achieves the slice genus. Suppose that K is smoothly slice and $tb_{\max}(K) = -1$. Does there exist a Legendrian knot Λ whose smooth knot type is K which bounds a Lagrangian disk? This is true for knots with ≤ 14 crossings. Danny Ruberman's comment: Joel Hass proved that a knot K is ribbon if and only if a representative within its isotopy class bounds an embedded minimal area disk. This problem seems to be related to the slice-ribbon conjecture. Matt Hedden's comment: Lagrangians are "through rigmarole" diffeomorphic to ribbon.

Problem 15 (Matt Hedden)**.** Suppose $tb(\Lambda) + rot(\Lambda) = 2g(\Lambda) - 1$. Note that if Λ is strongly quasipositive, it follows easily that the equality holds. Rudolph shows that if Λ is strongly quasipositive, then there exists an algebraic curve V in $\mathbb{C}^2$ such that $\Lambda \simeq \partial(V \cap B^4)$, and which satisfies $g(V \cap B^4) = g(\Lambda)$. (This is true for fibered knots.) By Rudolph, $\Lambda = \partial(V \cap B^4)$ for some algebraic curve V in $\mathbb{C}^2$ such that $g(V) = g(\Lambda)$. There is a slice-version of this question. Suppose that $tb(\Lambda) + rot(\Lambda) = 2g_4(\Lambda) - 1$. Is $\Lambda = \partial(V \cap B^4)$ for some algebraic curve V in B^4? (This is open for all knots.)Bounding an algebraic curve is equivalent, by work of Rudolph and Boileau-Orevkov, to being isotopic to the closure of a quasipositive braid.

Problem 16 (Matt Hedden)**.** By Eliashberg and Bennequin, a contact structure ξ on Y is tight if and only if $tb(\Lambda) + rot(\Lambda) \leq 2g(\Lambda) - 1$ for all Legendrian knots Λ. Is a contact structure ξ on Y tight if and only if $tb(\Lambda) + rot(\Lambda) \leq 2g_{Y\times I}(K) - 1$? In other words, is tightness characterized by the "slice–Bennequin inequality" holding in $Y \times [0,1]$? Of course the slice–Bennequin inequality implies the Bennequin inequality, so the question is whether tightness implies the slice–Bennequin inequality for $Y \times [0,1]$. By work of Hedden, it is known that the subset of tight contact structures with non-vanishing Ozsváth-Szabó contact invariant satisfy the $Y \times [0,1]$ slice–Bennequin inequality.

Problem 17 (Chris Davis)**.** Suppose that two winding number one patterns $P_1, P_2 \subset S^1 \times D^2$ satisfy that $P_1(K)$ and $P_2(K)$ are concordant for any knot K. Is P_1 and P_2 are concordant in $S^1 \times D^2 \times I$?

Problem 18 (Marco Golla)**.** Let Y be the Poincaré homology 3–sphere which is the boundary of the E_8 plumbing in Figure 2. What are the possible, minimal negative definite intersection forms on W such that $\partial W = Y$? (A negative definite intersection form $Q_W \colon H_2(W) \times H_2(W) \to \mathbb{Z}$ is *minimal* if there is no vector whose square is -1.) By Elkies and corrections terms, there is an explicit finite list of candidate minimal negative definite intersection forms. If Q_W is even, then Q_W is negative definite and hence $Q_W = E_8$. If Q_W is odd, then $H_1(W)$ is non-trivial and contains $\mathbb{Z}_4$–torsion by Frøyshov.

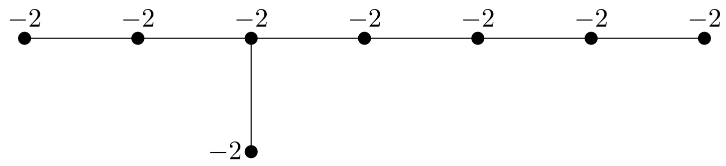

FIGURE 2. Plumbing diagram of E_8.