Research Statement

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My research lies in the field of low-dimensional topology, particularly knot theory and its applications towards the study of 3– and 4–manifolds. In this document I will describe my completed and current projects, and outline some future directions. Preprints of much of my work may be found at `http://arxiv.org/a/ray_a_1`.

Low-dimensional topology is the branch of topology which studies manifolds of dimension four and lower. Techniques which have yielded much information about manifolds of dimension five and higher often fail for 3– and 4–manifolds, and in fact, many specialized tools have been needed for studying these two particular cases. In some sense, one may consider dimension four as a boundary case between low and high dimensions: there are enough dimensions for the topology to exhibit complex behavior, but not enough space for our tools to work. This behavior is exemplified by the following: a closed manifold of dimension three or lower admits exactly one smooth structure; a closed manifold of dimension five or higher admits at most finitely many distinct smooth structures; however, a closed 4–manifold may have infinitely many distinct smooth structures.

A *link* is the image of a smooth embedding of a disjoint collection of circles into 3–space, considered up to isotopy; a *knot* is a link with a single component. The study of knots and links is intimately connected with the study of 3–manifolds as seen in the following famous theorem : any closed, connected, orientable 3–manifold can be obtained from the 3–sphere by performing a certain operation ('surgery') on some link. Just as the 3–dimensional relation of isotopy is related to the classification of 3–manifolds, there exist 4–dimensional relations on knots which are relevant to the classification of 4–manifolds.

My work so far has focused on these 4–dimensional equivalence relations known as concordance. Knots under concordance form the *knot concordance group*, denoted $\mathcal{C}$. Broadly speaking, my research aims to understand the structure of $\mathcal{C}$ using the following paradigms.

The action of satellite operators A reasonable approach towards studying any mathematical object is studying functions on it. In the case of knots, there is a natural choice of such functions, namely satellite operations, described in Figure 2. I study the action of satellite operators on the knot concordance group in [CDR12, DR13, Ray13c, Ray13b]. Satellite operations are also of independent interest beyond knot theory since they can be used to construct interesting examples of 3– and 4–manifolds.

Filtrations of the knot concordance group It is natural to seek to assess how 'close' a knot is to being trivial in $\mathcal{C}$, i.e. concordant to an unknotted circle. This notion was formalized when Cochran–Orr–Teichner introduced the *n–solvable filtration* of $\mathcal{C}$ and showed that the lower levels of the filtration encapsulate the information one can extract from various classical concordance invariants: in an almost quantifiable sense, the deeper a knot is within the n–solvable filtration, the closer it is to being trivial. Studying filtrations gives us a way of understanding the structure of $\mathcal{C}$, a large unwieldy object, in terms of smaller (and hopefully simpler) pieces. There are several other filtrations of knot concordance. In [Ray13a] I define a new filtration of $\mathcal{C}$ and establish relationships between the various filtrations.

In my work so far I have used tools from geometric and algebraic topology, contact geometry, Heegaard–Floer homology, and other techniques.

1. Background

A *knot* is the image of a smooth embedding $S^1 \hookrightarrow S^3$, considered up to isotopy. There is a natural operation on $\mathcal{K}$, the set of all knots, namely the *connected sum* operation, shown below.

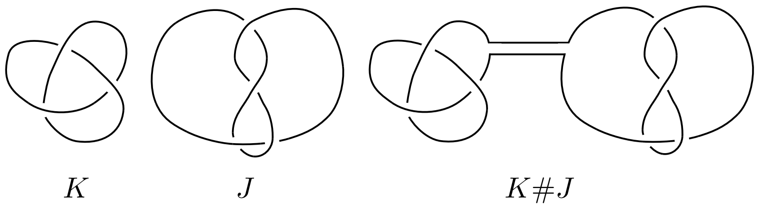

Figure 1. The connected sum operation on knots.

The connected sum operation can be generalized as follows. Given a knot P in a solid torus and any knot K, we obtain the *satellite knot* $P(K)$ by tying the solid torus into K, as shown in Figure 2. This is called the *satellite construction* under the *satellite operator* P.

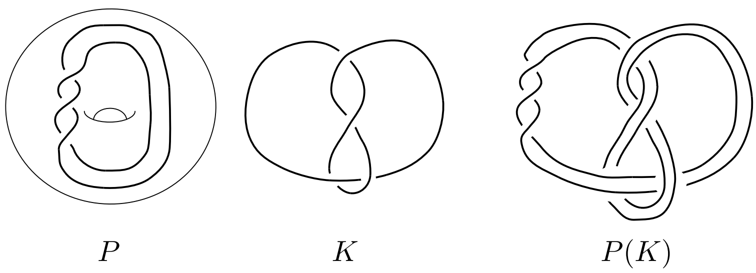

Figure 2. The satellite construction on knots.

Two knots $K_0 \hookrightarrow S^3 \times \{0\}$ and $K_1 \hookrightarrow S^3 \times \{1\}$ are said to be *concordant* if they cobound a smooth, properly embedded annulus in $S^3 \times [0,1]$. $\mathcal{K}$ modulo concordance forms an abelian group under connected sum, called the *knot concordance group*, denoted $\mathcal{C}$. Similarly, we say that two knots are *exotically concordant* if they cobound a smooth, properly embedded annulus in a smooth 4–manifold *homeomorphic* to $S^3 \times [0,1]$ (but not necessarily diffeomorphic). $\mathcal{K}$ modulo exotic concordance forms an abelian group called the *exotic knot concordance group*, denoted $\mathcal{C}^{\text{ex}}$. We can also work entirely in the topological category; we say that two knots are *topologically concordant* if they cobound a proper, topologically embedded, collared annulus in $S^3 \times [0,1]$. $\mathcal{K}$ modulo topological concordance forms an abelian group called the *topological knot concordance group*, denoted $\mathcal{C}^{\text{top}}$. Clearly we have that

$$\mathcal{C} \twoheadrightarrow \mathcal{C}^{\text{ex}} \twoheadrightarrow \mathcal{C}^{\text{top}}$$

If the 4–dimensional smooth Poincaré Conjecture is true, $\mathcal{C} = \mathcal{C}^{\text{ex}}$. There exist infinitely many knots which are topologically concordant to the unknot but not smoothly so (see, for example, [End95, Gom86, HK12, HLR12, Hom11]), i.e. the composed map above is known to have highly non-trivial kernel. For brevity, we will often use the notation $\mathcal{C}^*$ to denote $\mathcal{C}$, $\mathcal{C}^{\text{ex}}$, or $\mathcal{C}^{\text{top}}$.

The knots in the class of the unknot, under the various equivalence relations mentioned above, are important objects of study. A knot is called *slice* if it bounds a smooth, properly embedded disk in B^4, i.e. if it is concordant to the trivial knot. Similarly, a knot is *exotically slice* if it bounds a smooth, properly embedded disk in a smooth 4–manifold *homeomorphic* to B^4 (but not necessarily diffeomorphic). A knot is *topologically slice* if it bounds a proper, topologically embedded, collared disk in B^4.

2. Completed and current projects

2.1. **Slice knots which bound punctured Klein bottles.** Every knot K bounds an embedded, connected, oriented surface in S^3. If a slice knot K bounds a punctured torus F then, up to isotopy and orientation, there are exactly two homologically essential, simple, closed curves J_1 and J_2 on F with zero self-linking [Gil83]. If either J_i is slice, we can construct a slice disk for K by surgering F along J_i. Consequently, the curves J_i are called *surgery curves* for F. In 1982, Kauffman conjectured that a knot with a genus one Seifert surface F is slice if and only if F has a slice surgery curve [Kau87, Strong Conjecture, pp. 226]. While there was much evidence in the literature supporting this conjecture, such as in [CHL10, COT03, Coo82, Gil93], recently Cochran–Davis [CD13] have shown that Kauffman's conjecture is false.

The motivation for this project was to understand the analogous context of knots which bound punctured Klein bottles. If a knot K bounds a punctured Klein bottles F, we define the *longitude* λ of K to be a pushoff in the direction of F. Say that K bounds F with *zero framing* if, to parallel the orientable case, $\operatorname{lk}(K, \lambda) = 0$.

Theorem 2.1 (Propositions 3.4 and 3.5, and Theorem 5.4 of [Ray13c]). *Suppose a knot K bounds a punctured Klein bottle F with zero framing. Then, up to orientation and isotopy, there exists a unique 2–sided, homologically essential, simple, closed curve J embedded in F with self-linking zero. K is slice in a $\mathbb{Z}\left[\frac{1}{2}\right]$–homology 4–ball (and hence, rationally slice (i.e. slice in a $\mathbb{Q}$–homology B^4)) if and only if J is as well.*

Surgering F along the curve J in the above theorem yields a slice disk for K, and therefore we call it a surgery curve. Being rationally slice is a strong condition; several concordance invariants obstruct knots being $\mathbb{Q}$–concordant. For example, the Levine–Tristram signature function and the τ-invariant of Ozsváth–Szabó and Rasmussen [OS03, Ras03] are both zero for rationally slice knots. Therefore, our result shows that, in marked contrast to the punctured torus case, there are very strong restrictions on the concordance class of surgery curves on punctured Klein bottles.

Theorem 2.1 also has a surprising corollary about cable knots. The (p, q) cable of a knot K is obtained by applying the satellite operator consisting of the (p, q) torus knot to K.

Corollary 2.2 (Corollaries 5.5 and 5.6 of [Ray13c]). *Given knots K and J and any odd integer p, the $(2, p)$ cables of K and J are concordant in a $\mathbb{Z}\left[\frac{1}{2}\right]$–homology $S^3 \times [0,1]$ if and only if K is concordant to J in a $\mathbb{Z}\left[\frac{1}{2}\right]$–homology $S^3 \times [0,1]$. In particular, if the (2, 1) cable of K is slice in a $\mathbb{Z}\left[\frac{1}{2}\right]$–homology B^4, then K is slice in a $\mathbb{Z}\left[\frac{1}{2}\right]$–homology B^4.*

2.2. **The fractal nature of the knot concordance groups.** Any satellite operator P (i.e. a knot in a solid torus) gives a well-defined map from $\mathcal{C}^*$ to itself, by mapping the class of each knot K to the class of $P(K)$. Call such an operator *weakly injective* if $P(K) = P(U)$ implies $K = U$, and *injective* if $P(K) = P(J)$ implies $K = J$. Here U is the trivial knot and '=' denotes equivalence in $\mathcal{C}^*$. A long-standing open question asks if the Whitehead doubling operator is weakly injective on $\mathcal{C}$ [Kir97, Problem 1.38]. In [CHL11], several 'robust doubling operators' were introduced and evidence was provided for their injectivity but Corollary 2.2 was the first complete result in the realm of injectivity, albeit in terms of concordance in $\mathbb{Z}\left[\frac{1}{2}\right]$–homology $S^3 \times [0,1]$, i.e. '$\mathbb{Z}\left[\frac{1}{2}\right]$–concordance'. The following theorem greatly generalizes Corollary 2.2.

Theorem 2.3 (Theorem 5.1 of [CDR12]). *Any strong winding number one satellite operator P is injective on $\mathcal{C}^{ex}$ and $\mathcal{C}^{top}$. Any winding number n operator is injective on the group of $\mathbb{Z}\left[\frac{1}{n}\right]$-concordance classes of knots.*

The winding number of a satellite operator P in a solid torus V is the algebraic count of intersections between P and a generic meridional disk of V. For economy, we omit discussion of

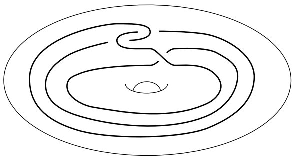

FIGURE 3. A strong winding number one satellite operator.

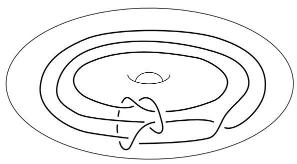

FIGURE 4. A bijective satellite operator.

'$\mathbb{Z}\left[\frac{1}{n}\right]$–concordance' and 'strong' winding number one. It suffices to know that there exist several strong winding number one operators and that any winding number one operator which is unknotted as a knot in S^3 is strong winding number one (for example, the operator in Figure 3.)

Theorem 2.3 is related to the possibility of $\mathcal{C}$ having a *fractal* structure. This was conjectured in [CHL11]. One may characterize 'fractalness' of a set as the existence of *self-similarities at arbitrarily small scales.* Theorem 2.3 shows that each strong winding number one satellite operator yields a self-similarity for $\mathcal{C}^{\text{ex}}$ and $\mathcal{C}^{\text{top}}$, but does not address the question of scale. For $\mathcal{C}^{\text{ex}}$, this is the objective of the following theorem.

Theorem 2.4 (Theorem A of [Ray13b]). *There exist infinitely many strong winding number one satellite operators P (such as the one shown in Figure 3) and a large class of knots K such that the knots $P^i(K)$ are distinct in $\mathcal{C}^{ex}$ and $\mathcal{C}$. That is, $P^i(K) \neq P^j(K)$ in $\mathcal{C}$ and $\mathcal{C}^{ex}$, for all $i \neq j \geq 0$.*

The action of the operator of Figure 3 on $\mathcal{C}^{\text{ex}}$ should be compared to the action of $f(x) = \frac{x}{3}$ on the Cantor ternary set, in that iterations give distinct images of $\mathcal{C}^{\text{ex}}$ at smaller and smaller scales. To complete the fractal analogy one must also address the question of *surjectivity* of strong winding number one operators; some progress towards this end has been achieved by Davis and the author in [DR13] (see Section 2.3).

Theorem 2.4 has some interesting applications. By choosing a topologically slice K, each operator P in the theorem yields an infinite family $\{P^i(K)\}$ of smooth (and exotic) concordance classes of topologically slice knots. Several such examples already exist in the literature (see [End95, Gom86, HK12, Hom11]; ours are novel only in the ease with which they are constructed and the added property that given any two knots in a family, one is a satellite of the other. Theorem 2.4 can also be used to construct infinite families of links with linking number one and unknotted components, which are each distinct from the class of the positive Hopf link.

2.3. **Satellite operators as group actions on knot concordance.** We recast the satellite operation on knot concordance classes, a monoid action, in terms of a group action by a larger set. Specifically, we study *generalized satellite operators* which form a subgroup of the group of homology cobordism classes of homology cylinders. This group was introduced by Levine in [Lev01]. We additionally show that the action of this subgroup on knots in homology 3–spheres is compatible with the classical satellite construction, and that any classical satellite operator gives a generalized satelite operator. More concretely, we prove a theorem of the following type. Let $\mathcal{S}^{\text{str}}$ denote the set of all strong winding number one satellite operators.

Theorem 2.5 (Main theorem of [DR13]). *For $\mathcal{C}^* = \mathcal{C}^{top}$ or $\mathcal{C}^{ex}$, there is an enlargement of $\mathcal{C}^*$, $\Psi : \mathcal{C}^* \hookrightarrow \widehat{\mathcal{C}^*}$, and a monoid morphism, $E : \mathcal{S}^{str} \to \widehat{\mathcal{S}^{str}}$, where $\widehat{\mathcal{S}^{str}}$ is a group which acts on $\widehat{\mathcal{C}^*}$ making the following diagram commute for all strong winding number one satellite operators P.*

$$\begin{array}{ccc} \mathcal{C}^* & \xrightarrow{P} & \mathcal{C}^* \\ \Psi \big\downarrow & & \big\downarrow \Psi \\ \widehat{\mathcal{C}^*} & \xrightarrow{E(P)} & \widehat{\mathcal{C}^*} \end{array}$$

Roughly, $\widehat{\mathcal{C}^*}$ is the set of all knots in homology spheres modulo a corresponding sense of concordance. Since $E(P)$ is an element of a *group* acting on $\widehat{\mathcal{C}^*}$, $E(P) : \widehat{\mathcal{C}^*} \to \widehat{\mathcal{C}^*}$ is a bijection. Theorem 2.3, which states that $P : \mathcal{C}^* \to \mathcal{C}^*$ is an injection, now follows from an elementary diagram chase.

Considering the satellite construction as given by a group action provides a novel approach to the problem of finding nontrivial *surjective* satellite operators on $\mathcal{C}^*$. While it is elementary to show that satellite operators with winding number other than ± 1 cannot give surjections on knot concordance [DR13, Proposition 4.5], very little is known in the case of satellite operators of winding number ± 1 other than the trivial connected-sum operators; a conjecture of Akbulut [Kir97, Problem 1.45] claiming that there exists a winding number one satellite operator P such that $P(K)$ is not slice for any knot K is wide open.

As an element of a group acting on $\widehat{\mathcal{C}^*}$, each strong winding number one satellite operator P gives a bijection $E(P) : \widehat{\mathcal{C}^*} \to \widehat{\mathcal{C}^*}$ with well-defined inverse $(E(P))^{-1}$. Instead of asking whether a knot K is in the image of P we may ask if $E(P)^{-1}(K) \in \widehat{\mathcal{C}^*}$ is in the image of $\Psi : \mathcal{C}^* \hookrightarrow \widehat{\mathcal{C}^*}$. This turns out to be an easier question to address and allows us to find a class of operators on $\mathcal{C}^*$ which are surjective (as well as injective). Such an operator is shown in Figure 4.

Theorem 2.5 also reveals a connection between the surjectivity of satellite operators and the conjectured existence of knots in homology 3–spheres which are not concordant to any knot in S^3. We connect Akbulut's conjecture mentioned above to a question of Matsumoto [Kir97, Problem 1.30] asking if every knot in a 3–manifold homology cobordant to S^3 is concordant to some knot in S^3 via a homology cobordism. We show that if Akbulut's conjecture is true then the answer to Matsumoto's question is no [DR13, Proposition 6.3].

2.4. **Casson towers and filtrations of the knot concordance group.** The n–solvable filtration [COT03] has been instrumental in the study of (smooth and topological) knot concordance in recent years. Part of the justification for the naturality of the n–solvable filtration is its close relationships with several more geometric filtrations of $\mathcal{C}$, as revealed in the following theorem.

Theorem 2.6 (Theorems 8.11 and 8.12 of [COT03])**.** *If a knot K bounds a grope of height $n+2$, then K is n–solvable. If a knot K bounds a Whitney tower of height $n+2$, then K is n–solvable.*

Cochran–Harvey–Horn [CHH13] have recently introduced a new pair of filtrations of $\mathcal{C}$, the *positive* and *negative* filtrations: $\{\mathcal{P}_n\}_{n=0}^{\infty}$ and $\{\mathcal{N}_n\}_{n=0}^{\infty}$ respectively. These new filtrations are of interest because (unlike the n–solvable filtration) they can be used to study smooth concordance classes of topologically slice knots. In [Ray13a], I prove counterparts of Theorem 2.6 for the positive and negative filtrations in terms of *Casson towers* [Cas86, Fre82]. A Casson tower of height one is a regular neighborhood of a disk with transverse self-intersections. Each Casson tower T has a canonical set of curves in ∂T generating $\pi_1(T)$, called a *standard set of curves* for T. A Casson tower of height n is obtained by attaching Casson towers of height one to a Casson tower of height $n-1$ along the standard set of generators. A schematic picture is given in Figure 5. Every Casson tower T has a 2–complex as a strong deformation retract, called its *core*.

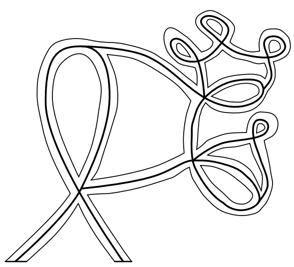

Figure 5. Schematic diagram of a Casson tower of height three.

If a knot K bounds a kinky disk in B^4 with only positive (resp. negative) kinks, $K \in \mathcal{P}_0$ (resp. $K \in \mathcal{N}_0$). Since kinky disks are closely related to the zero'th level of the positive and negative filtrations, Casson towers—built using layers of kinky disks—are natural objects to study in this context. The following definitions generalize the notion of 'bounding a kinky disk with only positive (resp. negative) kinks'.

Definition 2.7 (Definitions 1–4 of [Ray13a])**.** *For any knot K,*

(1) *$K \in \mathfrak{C}_n$ if it bounds a Casson tower of height n in B^4. $K \in \mathfrak{C}_n^+$ (resp. $\mathfrak{C}_n^-$) if it bounds a Casson tower of height n in B^4 such that the base-level kinks are all positive (resp. negative).*

(2) *$K \in \mathfrak{C}_{2,n}$ if it bounds a Casson tower T of height two in B^4 such that each member of a standard set of curves for T is in $\pi_1(B^4 - C)^{(n)}$, where C is the core of T. $K \in \mathfrak{C}_{2,n}^+$ (resp. $\mathfrak{C}_{2,n}^-$) if it bounds a Casson tower T of height two in B^4 such that all the base-level kinks are positive (resp. negative) and each member of a standard set of curves for T is in $\pi_1(B^4 - C)^{(n)}$, where C is the core of T.*

We establish the following set of relationships between various filtrations of $\mathcal{C}$.

Theorem 2.8 (Theorem A from [Ray13a])**.** *Let $\{\mathcal{F}_n\}_{n=0}^\infty$ denote the n–solvable filtration of $\mathcal{C}$ and $\{\mathcal{G}_n\}_{n=0}^\infty$ the (symmetric) grope filtration of $\mathcal{C}$. For any $n \geq 0$,*

(i) $\mathfrak{C}_{n+2} \subseteq \mathcal{G}_{n+2} \subseteq \mathcal{F}_n$,
(ii) $\mathfrak{C}_{2,n} \subseteq \mathcal{F}_n$,
(iii) $\mathfrak{C}_{n+2}^+ \subseteq \mathfrak{C}_{2,n}^+ \subseteq \mathcal{P}_n$,
(iv) $\mathfrak{C}_{n+2}^- \subseteq \mathfrak{C}_{2,n}^- \subseteq \mathcal{N}_n$.

The second inclusion in part (i) is exactly the second result listed earlier in Theorem 1 [COT03, Theorem 8.11]; we include it here for completeness.

By combining several theorems from 4–manifold topology ([Fre82, Theorems 1.1 and 4.4][Gom05, Proposition 5.2][GS84, Theorem 5.1][Qui82, Proposition 2.2.4]) we can see that $\mathfrak{C}_5$ is *equal* to the set of all topologically slice knots. It is conjectured that $\mathfrak{C}_3$ is equal to the set of topologically slice knots. As a result, it is natural to study $\{\mathfrak{C}_{2,n}\}_{n=0}^\infty$, to filter between $\mathfrak{C}_2$ and $\mathfrak{C}_3$. Theorem 2.8 indicates that these filtrations are a worthwhile object of study and in the case of links, the $\{\mathfrak{C}_{2,n}\}_{n=0}^\infty$ filtration is smaller than the n–solvable filtration as follows.

Proposition 2.9 (Proposition 6.3 of [Ray13a])**.** *For m–component links, let $\mathfrak{C}_{2,n}(m)$ and $\mathcal{F}_n(m)$ denote the Casson tower and n–solvable filtrations respectively. For all n, $\mathfrak{C}_{2,n}(m) \subsetneq \mathcal{F}_n(m)$ for $m \geq 2^{n+2}$.*

By showing that $\mathfrak{C}_3 \subseteq \bigcap \mathcal{F}_n$, we infer that either every knot in $\mathfrak{C}_3$ is topologically slice or there exist knots in $\bigcap \mathcal{F}_n$ which are not topologically slice. The only presently known elements of $\bigcap \mathcal{F}_n$ are topologically slice knots and it is an open question whether *all* knots in $\bigcap \mathcal{F}_n$ are topologically slice.

While any topologically slice knot K bounds an arbitrarily tall Casson tower, not all of them are even in $\mathfrak{C}_1^\pm$. For links of two or more components, each level of the positive and negative filtrations is non-trivial [CP12]. Mirroring the fact that the positive and negative filtrations non-trivially filter topologically slice knots and links, it is expected that the filtrations $\{\mathfrak{C}_{2,n}^\pm\}$ will as well.

3. Future directions

3.1. **Shake concordance of knots.** (Joint project with Tim Cochran) For any knot K, define an *algebraically one collection* to be a collection of $2n+1$ 0–framed parallels of K, where $n+1$ of the parallels are oriented in the direction of K and n are oriented in the opposite direction, for some $n \geq 0$. Knots K_0 and K_1 are said to be *shake concordant* if there is a smooth, properly embedded genus zero surface A in $S^3 \times [0,1]$, where $A \cap S^3 \times \{0\}$ is an algebraically one collection of K_0

and $A \cap S^3 \times \{1\}$ is an algebraically one collection of K_1. This is clearly a generalization of knot concordance.

We have the following result at present.

Theorem 3.1 (Cochran–R.)**.** *There exist infinitely many knots (even topologically slice knots) that are shake concordant but not concordant. 4–ball genus, Ozsváth–Szabó's τ–invariant, and the s–invariant from Khovanov homology all fail to be invariants of shake concordance.*

3.2. **The structure of $\mathcal{C}^{\text{ex}}$.** If the 4–dimensional smooth Poincaré Conjecture is true, then $\mathcal{C} = \mathcal{C}^{\text{ex}}$. Recall that $\mathcal{C}$ is the group obtained by identifying knots concordant in a $S^3 \times [0,1]$ whereas $\mathcal{C}^{\text{ex}}$ is obtained by identifying knots concordant in a potentially exotic $S^3 \times [0,1]$. A closed, simply connected 4–manifold differs from an exotic copy only by single *Akbulut cork* [AM98, CFHS96, Mat96]. I am interested in applying the theory of Akbulut corks to study the difference between $\mathcal{C}$ and $\mathcal{C}^{\text{ex}}$. It seems likely that the groups are the same. However, if they are not, one may infer that the smooth 4–dimensional smooth Poincaré Conjecture is false!

3.3. **Surjectivity of satellite operators.** [DR13] gives a novel approach towards finding surjective satellite operators, or proving that a given satellite operator is not surjective. I am interested in pursuing this further, utilizing tools from contact topology and Heegaard–Floer homology. The existence of a non-surjective strong winding number one satellite operator would complete the fractal analogy for $\mathcal{C}^*$ as mentioned in Section 2.2. It would also address long-standing questions of Akbulut and Matsumoto [Kir97, Problems 1.30 and 1.45]; that is, as shown in Section 2.3 and [DR13], if there exists a winding number one satellite operator whose image does not include the unknot then there exists a knot in a 3–manifold homology cobordant to S^3 which is not concordant to *any* knot in S^3 via a homology cobordism.

3.4. **Relationship between winding number one satellite operators and Akbulut corks.** Several of the winding number one operators considered in [Ray13b] also show up in the literature relating to Akbulut corks—contractible 4–manifolds endowed with an involution on its boundary which extends to a self-homemorphism but not a self-diffeomorphism—such as in [AY08]. Akbulut corks are closely related to the existence of exotic smooth structures on 4–manifolds. I am interested in understanding how satellite operators and Akbulut corks are related and whether one may apply results about satellite operators, such as those in Section 2.2, to the study of exotic smooth structures on 4–manifolds.

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