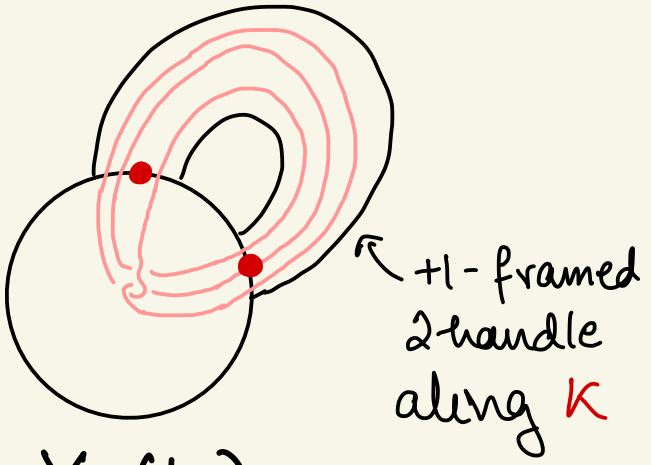
$K \subseteq S^3$ is ± 1-shake slice iff $\mathrm{Arf}(K) = 0$

Feller - Miller - Nagel - Orson - Powell - R.

$(\Longrightarrow)$ [Robertello 1965]

$(\Longleftarrow)$ Suppose $\mathrm{Arf}(K) = 0$



$X_1(K)$

$K \subseteq S^3$ is 1-shake slice if a gen of $H_2(X_1(K); \mathbb{Z}) \cong \mathbb{Z}$ is represented by an embedded sphere

[Freedman 1982] $S^3_1(K) = \partial C^4$.

C compact, contractible

[Boyer 1986] $C \# \mathbb{CP}^2 \cong X_1(K)$ iff $ks(C \# \mathbb{CP}^2) = 0$

$\cong$: homeomorphic

ks: Kirby-Siebenmann invariant

$$X_1(K) \cong C \# \mathbb{CP}^2 \Longrightarrow K \text{ 1-shake slice.}$$

$$\begin{aligned} Rs(C \# \mathbb{CP}^2) &= Rs(C) + Rs(\mathbb{CP}^2) \qquad \text{additive under \#} \\ &= Rs(C) \qquad Rs = 0 \text{ for sm. mflds} \\ &= \mu(S^3_1(K)) \qquad \text{[see Freedman-Quinn 1990]} \\ &\quad \uparrow \text{Rochlin invariant} \\ &= Arf(K) \qquad \text{[González-Acuña 1970]} \\ &= 0 \end{aligned}$$

□