Shake slice and shake concordant knots

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Representing homology classes

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For a 4–manifold X, given $\alpha \in H_2(X;\mathbb{Z})$, can we represent α by an embedded sphere?

This is related to the minimal genus question, i.e. given α, what is the minimal genus of a surface representative of α?

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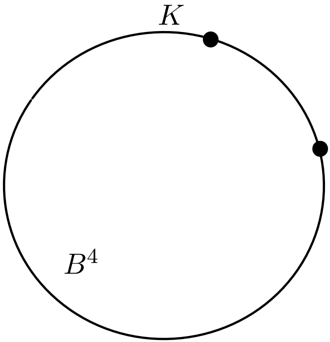

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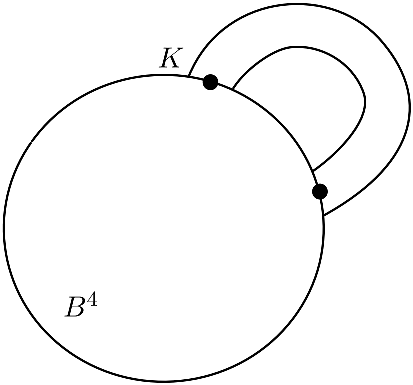

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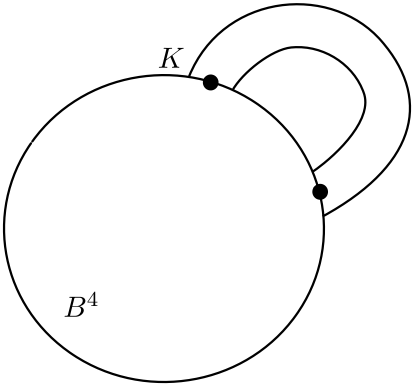

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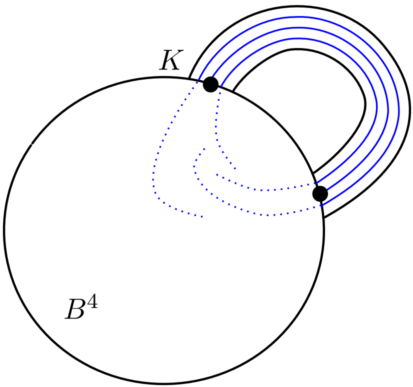

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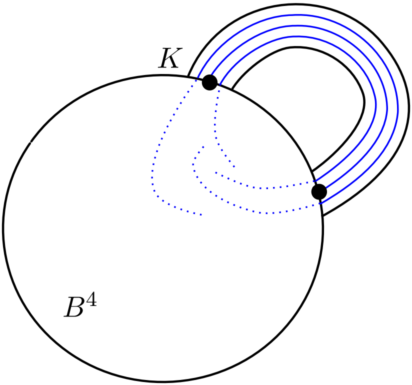

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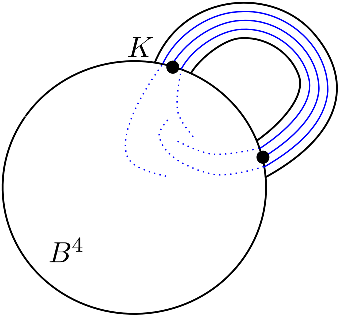
Not all knots are shake slice (Akbulut).

Shake slice knots



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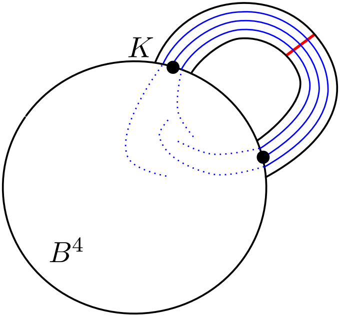

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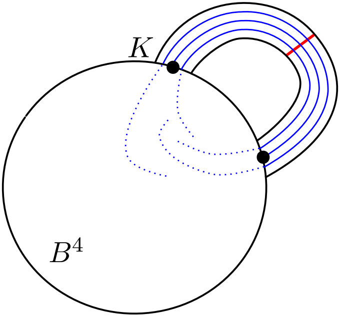

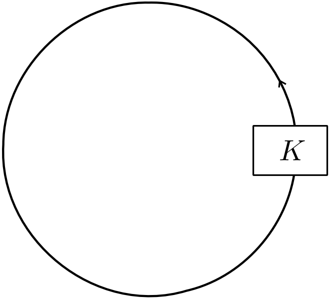

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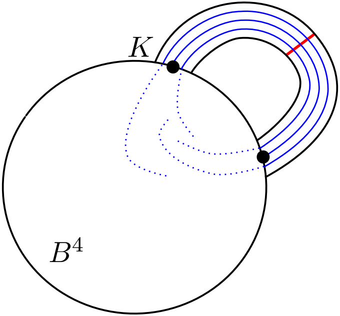

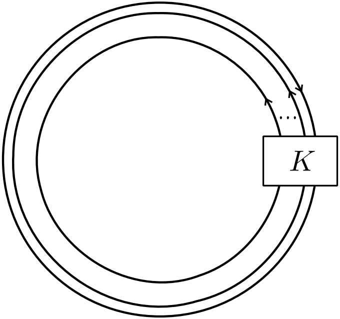

Figure: A shaking of the knot K

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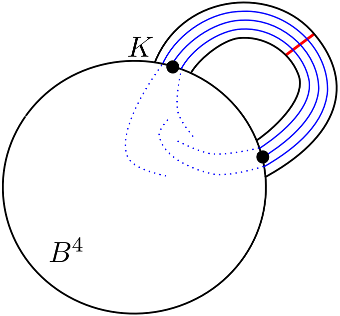

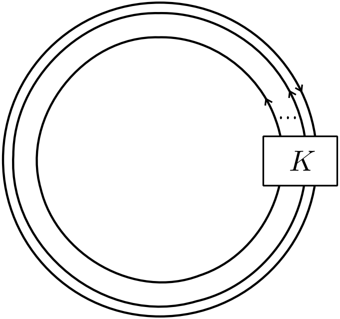

Figure: A shaking of the knot K

Proposition (Cochran–R.)

A knot K is shake slice if and only if some shaking of K bounds a genus zero surface in B^4.

Slice knots and shake slice knots

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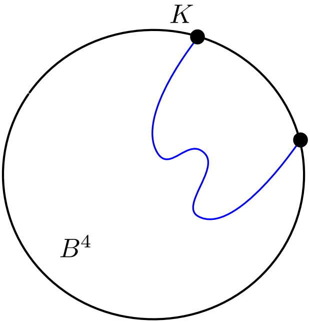
A knot K in S^3 is said to be slice if it bounds a disk in B^4.



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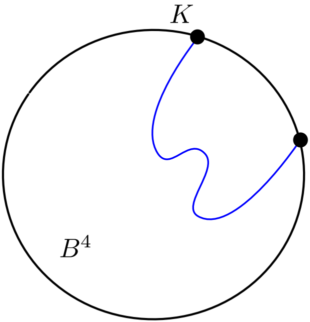
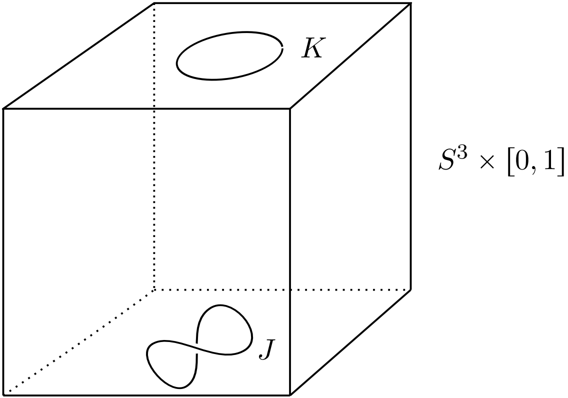


If K is slice, it is shake slice. The converse is open (since 1977).

Concordance of knots



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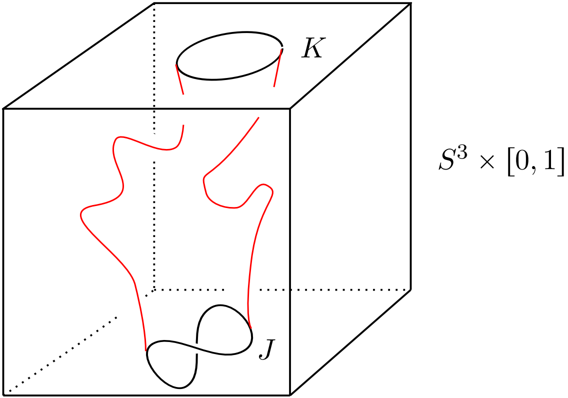

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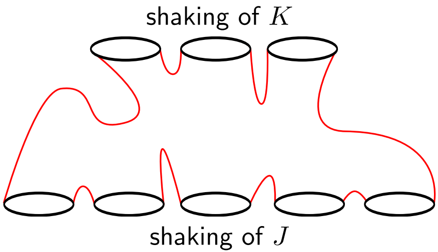

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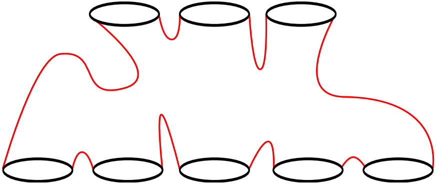

Question

Are there knots that are shake concordant but not concordant?

Shake concordance and 0–surgery manifolds

For any knot K, let M_K denote the manifold obtained by performing 0–framed surgery on S^3 along K.

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Cochran–Franklin–Hedden–Horn

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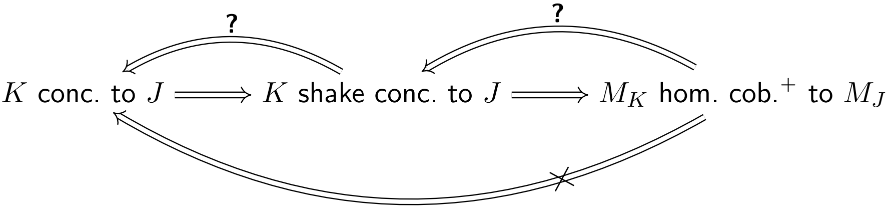

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Results

Theorem (Cochran–R.)

There exist infinitely many (topologically slice) knots that are distinct in concordance but are pairwise shake concordant.

In addition, τ, s, and slice genus all fail to be invariants of shake concordance.

Results

The previous result follows from a characterization theorem for shake concordant knots.

Theorem (Cochran–R.)

K is shake concordant to J if and only if there exist winding number one patterns P, Q, with $P(U), Q(U)$ slice such that $P(K)$ is concordant to $Q(J)$.

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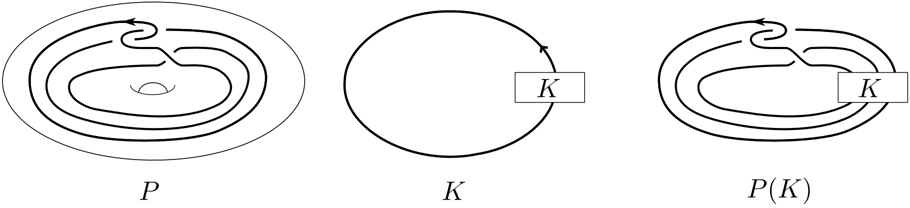

Figure: The satellite operation on knots

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Corollary (Cochran–R.)

The equivalence relation on the set of isotopy classes of knots generated by shake concordance is the same as the one generated by concordance and setting a knot equal to its satellites under slice winding number one patterns.

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We also get a characterization of shake slice knots.

Corollary (Cochran–R.)

K is shake slice if and only if there exists a winding number one pattern P such that $P(U)$ and $P(K)$ are slice.

This follows from the characterization theorem, since a knot is shake slice if and only if it is shake concordant to the unknot.