Homework A1 due Dec 11

1(a) Show that isotopy of the attaching map does not change the diffeomorphism type of the resulting space.
i.e. if X is a manifold, h is a k-handle and the maps $\varphi, \varphi' : \partial D^k \times D^{n-k}$ are isotopic, then $X \cup_\varphi h$ and $X \cup_{\varphi'} h$ are diffeomorphic.

Hint: isotopy extension theorem.

(b) Show that if X^n is compact and connected, then $(X, \partial_- X)$ admits a handle decomp. with a single 0-handle (if $\partial_- X = \emptyset$) or no 0-handles (if $\partial_- X \neq \emptyset$). Formulate a similar statement for n-handles. What happens if X is not connected? Not compact?

(c) Show that a handle decomposition can be modified so that handles are attached in increasing order of index (handles of the same index may be attached in any order, including simultaneously).

Hint: transversality.

2(a) Give a handle decomposition for $\mathbb{RP}(2)$ and the Klein bottle.
(b) Give a handle decomposition for $S^1 \times S^2$.
(c) Give a handle decomposition for $S^1 \times S^3$.

Don't forget to draw pictures!

3(a) Show that any lens space can be described by a pair (p,q) of relatively prime integers.

Hint: HW7, class.

(b) Give the handle diagrams for $L(5,1)$ and $L(5,2)$.
(c) Use the above to compute π_1, H_*, H^*.
(d) For each $n \geqslant 0$, construct a 3-manifold with $\pi_1 \cong \mathbb{Z}/n$.

Fun (optional thing to think about / look up later if interested): When are two lens spaces homeomorphic? Homotopy equivalent?