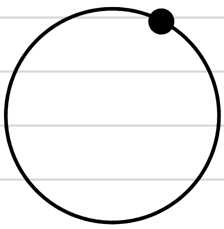
Homework A2

Optional / Bonus

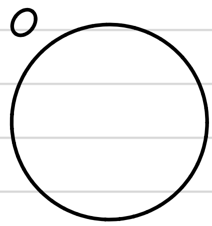
1. Show that surgery on the S^1 defined by a 1-handle corresponds to removing the dot from the dotted circle and replacing it with a zero.

By surgery, I mean remove a tubular neighbourhood $S^1 \times D^3$ and glue in a copy of $D^2 \times S^2$, using the trivial framing.

Note in particular that the boundary of the 4-mfld does not change. i.e.



and



have the same boundary.

2 a) Show that $S^2 \tilde{\times} S^2$ and $\mathbb{CP}(2) \# \overline{\mathbb{CP}(2)}$ are diffeomorphic.

b) Show that $S^2 \times S^2 \# \mathbb{CP}(2) \cong \mathbb{CP}(2) \# \mathbb{CP}(2) \# \overline{\mathbb{CP}(2)}$.

3 a) Show that any finitely presented group G can be realised as the fundamental group of a closed, oriented 4-manifold.

b) Let X be a closed, orientable 4-mfld. Prove that the intersection form Q_X is unimodular i.e. any matrix representing the intersection form has determinant ± 1. Show the same is true when ∂X is an integer homology sphere, i.e. $H_*(\partial X) \cong H_*(S^3)$.

4. What are the closed, oriented 4-mflds that can be obtained from a Kirby diagram consisting of a Hopf link (no 1- or 3-handles)?