Homework #3 — due Jan 14

1. For a compact oriented n-manifold X, define the **double** of X to be

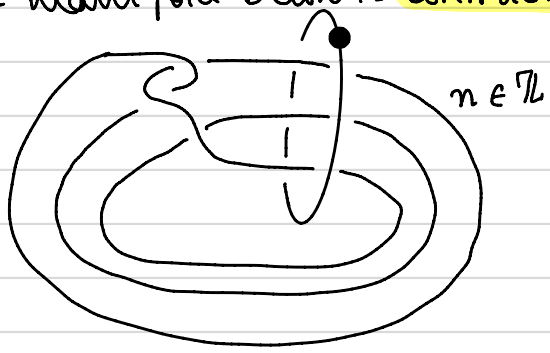
$$DX = X \underset{\partial X}{\cup} \bar{X}$$

where $\bar{X}$ is X with the reversed orientation, and X & $\bar{X}$ are glued using the identity map on $\partial X = \partial \bar{X}$ (as unoriented mflds).

Let X be a 4-mfld with no 3- or 4-handles. Given a Kirby diagram for X, describe how to get a Kirby diagram for DX.

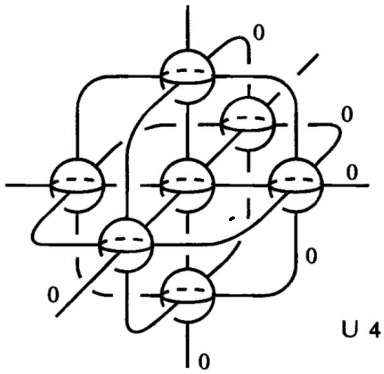
Hint: the double of $S^2 \times D^2$ is $S^2 \times S^2$. How are their Kirby diagrams related? Turn the handles of $\bar{X}$ upside down.

Show that the manifold below is **contractible** and that its double is S^4. for any $n \in \mathbb{Z}$



When $n=0$, this is called the **Akbulut cork**

2. We saw in class that the diagram on the right gives the 4-torus $T^4 = S^1 \times S^1 \times S^1 \times S^1$. Find four ways to identify this with $S^1 \times T^3$. Find six (different) copies of T^2 in the figure.



3. We saw that the diagram on the right gives the 3-torus $T^3 = S^1 \times S^1 \times S^1$. Find three (different) copies of T^2 in the figure. Find two tori that intersect transversely in a homologically nontrivial circle, and three tori that intersect in a single point