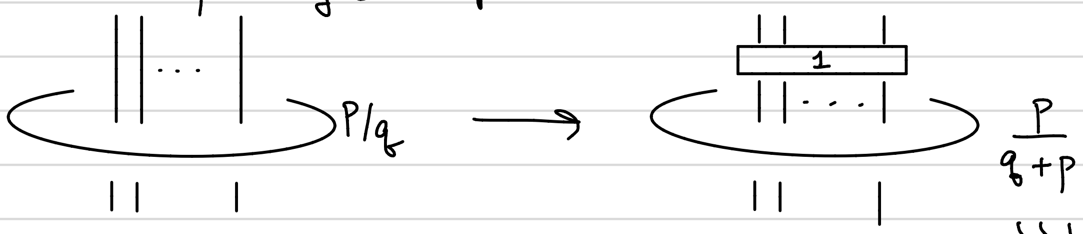
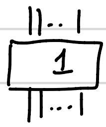
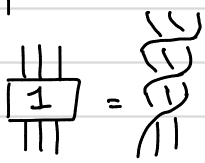
Homework A4 due Jan 21

1 (a) Prove that the following move on a Dehn surgery diagram preserves the diffeo. type of the corresponding 3-manifold.

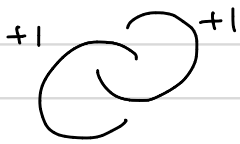


where  denotes one full positive twist, i.e. 

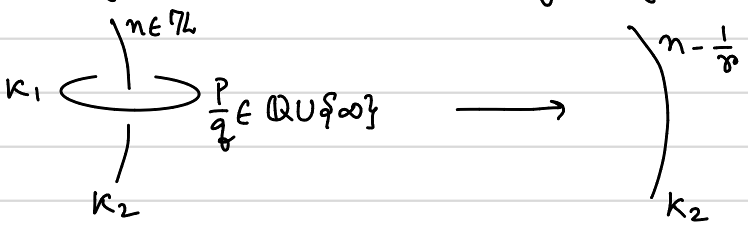
In other words, we can find an unknotted component U in the diagram and twist the strands going through it at the expense of changing the framing on U.
Hint: apply a Dehn twist to a solid torus.

b) How does the framing on the other components change?

The above process is called a Rolfsen twist

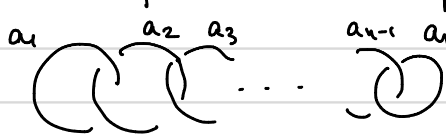
c) Use a Rolfsen twist to see that  describes $S^2 \times S^1$.

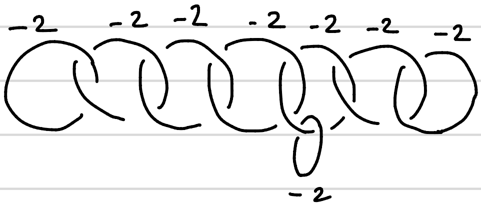
2 The following move on Dehn surgery diagrams is called a slam dunk



Fact: Slam dunk preserves the diffeo type of the 3 mfld.

a) Let a_i be integers. Show that the diagram below represents a lens space $L(p,q)$. What is the relationship between $\{a_i\}$ and $\{p,q\}$?



b) Show that  describes a Dehn surgery on a trefoil knot.

Hint: feel free to use Rolfsen twists, blow up/down

c) For any knot K and rel. prime integers (p,q), compute $H_*(S^3_{p/q}(K);\mathbb{Z})$, where $S^3_{p/q}(K)$ is the result of Dehn surgery on S^3 along K with framing p/q.
When is $H_*(S^3_{p/q}(K)) \cong H_*(S^3)$? When is $H_*(S^3_{p/q}(K)) \cong H_*(S^1 \times S^2)$?