

## MATH 23B-1: INTRODUCTION TO PROOFS SPRING 2016

**Instructor:** Arunima Ray (feel free to call me Aru)

**Email:** aruray@brandeis.edu

**Course website:** LATTE, Piazza

**Lecture time:** MWTh 9a–950a (Block B)

**Office:** Goldsmith 205

**Office hours:** W 5-6p, Th 11a-noon, and by appointment

**Teaching Assistants:** TBA

**TA office hours:** TBA

**Prerequisites:** Math 15a, 20a, or 22a, or permission of the instructor. Please come see me if you have any concerns about whether you have the right mathematical experience for this course.

**Textbook:** *A discrete transition to advanced mathematics*, Richmond and Richmond, American Mathematical Society, ISBN: 978-0-8218-4789-3

**Exemption exam:** A few students each year have strong enough mathematical backgrounds that they do not need to take Math 23b. We offer the Math 23b exemption exam for them. For more information, see <http://people.brandeis.edu/~charney/23bex.html>

If you would like to attempt the exemption exam, please get in touch with me immediately.

**Goals:** The foundations of education consists of three R's – Reading, wRiting, and aRithmetic. This course will address all three. We will spend a lot of time Reading our textbook, and wRiting clear and logical arguments; the aRithmetic, or rather mathematics, that we will learn will mostly be the means towards achieving these other goals.

This course is a bridge between your introductory mathematics courses, which are based primarily on computations, and advanced mathematics courses, which are based primarily on proofs. Many people refer to mathematics as a language – in this class you will begin to learn to speak this language. We will start with the most fundamental concepts and grammar rules. After we have some familiarity with the language of formal mathematics, we will practice this language in the setting of discrete mathematics. More like an advanced language class, merely memorizing the vocabulary will not suffice (in fact, hopefully we can keep vocabulary to a minimum), but rather you will be required to understand and speak clearly in this language. The material learned here will help you understand the mathematics you read and clarify the mathematics you write. Because we are learning how to write mathematics, exposition will also be a component in your evaluation.

In terms of content we will mostly cover Chapters 1–3, 5–6, 8 in our textbook.

Through this course, students will:

- (1) develop familiarity with reading mathematics independently, understanding abstract definitions and using supporting examples and counterexamples,
- (2) apply the logical structure of proofs and work symbolically with connectives and quantifiers to produce logically valid, correct and clear arguments,
- (3) recognize direct and indirect proofs, as well as proofs by induction
- (4) perform set operations on finite and infinite collections of sets and be familiar with properties of set operations,
- (5) define and illustrate the concepts of relations and functions, particularly injections, surjections, direct and inverse images and inverse functions,
- (6) recognize and construct equivalence relations on sets and equivalence classes,
- (7) critically analyze mathematical arguments with respect to logic and correctness,
- (8) write solutions to problems and proofs of theorems that meet rigorous standards based on content, organization and coherence, argument and support, and style and mechanics.

**Grading:** Your grade will be calculated as follows (the scheme which maximizes your personal grade will be used).

Item	Scheme 1	Scheme 2
Group/class work	15%	15%
Individual problem sets	30%	30%
Participation	5%	5%
Midterm	25%	15%
Essay	10%	10%
Final exam	15%	25%

If there are any concerns about grading, please see me within one week of getting the assignment/exam back, and before the last day of class (May 2). There might be a curve at the end of the semester based on student performance.

There will be several extra-credit opportunities (see below).

**Course structure:** I am thinking of this course as a lab course. Most of our classwork will be hands-on and in groups. Mondays will be mostly lecture with some group work, Wednesdays and Thursdays will be mostly group work with some lecture. This will be adjusted if we miss classes for holidays/snow days/etc.

Bring your textbook to class.

**Group work:** The first set of groups will be created on Jan 20. A new set of groups will be created on Mar 7. In class, you will mostly work within your groups on group assignments. These assignments are designed to be completed in class itself. In case you would like extra time to finish your group assignment, you can take them home – group assignments will almost always be due at the beginning of class on Mondays (see schedule). There will be 10 group assignments in total.

If you miss any class days, you should clear this with your group ahead of time; together you should determine how you can make up for your absence.

I will be conducting periodic surveys to ensure that each group-member is contributing to their group to the best of their ability; if you are not, you may receive only a fraction of the points your group has earned.

**Individual problem sets:** There will be six individual problem sets (see below for schedule).

Problem sets will consist primarily of writing proofs. These should be written in complete sentences – exposition will be a large part of your grade. You will have the opportunity to rewrite the hardest proofs.

You are welcome to work with your groupmates or other classmates on problem sets, but your final writeup must be your own!! In particular, you are not permitted to look up solutions to the problems in any written form, including on the internet. If in doubt about what sources you can use, ask me.

**Late work policy:** Life tends to get in the way of classes sometimes. To help with this, you will have some “late days”. A late day can be applied to submit your work late by at most one day (24 hours) without penalty (I might ask you to submit your work via email if I happen to be out of town or it is a weekend). You are welcome to use multiple late days on the same assignment.

Each student will have **six** late days to be used for the individual problem sets. Each group will have **two** late days for group assignments.

To apply a late day, you must talk to me in person, or send me an email/private message on Piazza clearly stating that you want to use a late day.

If you have chosen not to apply a late day, or have run out of them, I will deduct  $\frac{1}{7}$  of the assignment grade per day it is late. E.g. if you submit a 100-pt assignment two days late and receive a grade of 70, you will receive a score of 50.

**Participation:** Your participation grade will be based on your participation in the following:

- (1) in-class activities, including sincerely working with your group, asking/answering questions, etc.,
- (2) discussion forum: post at least five sincere (non-anonymous) questions, comments or answers on Piazza.
- (3) reading assignment: there will be a short reading assignment due every Sunday night at 8p. These will be posted on the course **notabene** site. See LATTE for enrollment instructions. Post at least five constructive comments.
- (4) reflection activities: respond to periodic reflection prompts (graded on completion).
- (5) essay contest evaluation: participate in the voting for the essay contest (graded on competition).

**Midterm:** The midterm will be in-class during our usual class time on Mar 3. There will be no make-up exams without prior approval and an appropriate reason. The exam will be closed-book, closed-notes, closed-friends, and open-brain.

**Essay:** The essay will be based on a mathematical topic of your choice. The intended audience consists of your classmates - the essay should be related to mathematics but expository in nature, e.g. you might think of it as a blogpost.

The essay should be one-page, typed (margins 1–1.5in, font 10–12pt (figure captions and references 8-12pt)), and can contain pictures and figures. A submission containing any offensive material will be immediately disqualified and you will receive a score of zero for the assignment.

It might be useful to type up your essay in  $\text{\LaTeX}$ , but you are welcome to use any other typesetting software, such as Word, OpenOffice, GoogleDocs, etc.

You are welcome to talk to anyone you like about your essay (including the Writing Center in the library) as long as it is written in your own words.

The essay will be graded as follows

Submitted essay topic on time	1 point
Submitted first draft on time	1 point
Essay content	4 points
Essay exposition	4 points

We will hold a contest for the essays - voting in this contest is part of your participation grade. (Names will be removed from essays when voting occurs). The finalists will receive bonus points on their final grade, and the eventual winner will receive an additional prize!

Some possible topics: fractals, equilibrium in economical systems, mathematics in music, a mathematician of your choice, such as Pythagoras, Noether, Galois, Kovalevsky, Wiles, Mirzakhani, etc., growth of populations, the Mobius strip, the Klein bottle, your favorite mathematical puzzle, the Fibonacci sequence, a particular application of a mathematical result, Schrodinger's equation, the book Flatland, imaginary numbers, complex numbers and how they are used in physics, your favorite mathematical proof, etc.

My favorite math blog is <http://blogs.scientificamerican.com/roots-of-unity/>, I recommend starting there, clicking lots of links and googling lots of things.

You are welcome to use a topic from our textbook, but if it is a topic that we discuss in class, you will be graded more strictly on the content.

**Extra credit:** You can earn up to five points on your final grade in the following ways.

- (1) Be a finalist in the essay contest (see above)
- (2) Participate in a mathematical activity, such as visit the math exhibit at the Museum of Science, or the Museum of Math in NYC, or a Putnam practice session here, or some sort of public presentation in mathematics. I will periodically announce such

opportunities; if you have an activity in mind, make sure to ask me if it is applicable. To receive credit, you will have to write a (very short) report on the activity.

- (3) Create or substantially edit a wikipedia page about a math topic/individual. (If editing an article, run it by me to make sure the edits are substantial enough.)
- (4) Write a publicly available blogpost about a math topic of your choice (this could consist of making your essay publicly available - please don't do this until after the essay contest has ended).
- (5) I'm open to other possibilities - if you think you should get extra credit for something, let me know and there's a good chance I'll agree with you.

The above will be graded primarily on completion – i.e. if you complete the activity in good faith, you will get your points.

**Discussions:** We will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from classmates, the TA, and myself. Rather than emailing questions to me, I encourage you to post your questions on Piazza. Find our class page at: <https://piazza.com/brandeis/spring2016/math23b1/home>

**Feedback:** If you are at all concerned about anything related to our course (or anything else), please feel free to send me an email, a message on Piazza, or just drop by my office. Please remember, the sooner you tell me your concerns, the more I can do about them. In addition, you can send me anonymous feedback through the google form available at <http://goo.gl/forms/HUHQaWRDXk> (this link can also be found on LATTE/Piazza).

**Expectations:** Students are expected to complete readings on time, attend class, work sincerely with their groupmates to the best of their ability. Success in this 4 credit hour course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for class (readings, papers, discussion sections, preparation for exams, etc.).

In return, my goal is for each and every one of you to do well in this course, in the sense of learning something new and fun, and also getting a good grade.

**Disability support:** If you are a student with a documented disability on record at Brandeis University and wish to have a reasonable accommodation made for you in this class, please see me immediately.

**Academic integrity:** You are expected to be familiar with, and to follow, the University's policies on academic integrity. Please consult Brandeis University Rights and Responsibilities for all policies and procedures. All policies related to academic integrity apply to in-class and take home projects, assignments, exams, and quizzes. Students may only collaborate on assignments with permission from the instructor. Allegations of alleged academic dishonesty will be forwarded to the Director of Academic Integrity. Sanctions for academic dishonesty can include failing grades and/or suspension from the university.

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**TL;DR:**

- (1) There is a reading assignment due on **notabene** every Sunday at 8p. Comment on these for participation points
- (2) Mondays will be mostly lecture, Wednesdays and Thursdays will be mostly group work.
- (3) Groups will turn in their class work roughly every week.
- (4) There is an individual problem set due roughly every two weeks.
- (5) There is one in-class midterm in early March.
- (6) There is a one-page paper due in early April. Finalists get extra credit, winner gets additional prize. Vote in the contest for participation points.
- (7) Participate in the discussion forum on Piazza to get help on questions and for participation points.
- (8) There are many extra credit opportunities.
- (9) Come talk to me if you have any concerns.

**Course Schedule:**

Groups created	Jan 20
GW1, <b>PS1</b>	Jan 25
GW2	Feb 1
GW3, <b>PS2</b>	Feb 8
GW4, <b>PS3</b>	Feb 22
GW5	Feb 29
<b>Midterm</b>	Mar 3
New groups created	Mar 7
Essay topics due, GW6	Mar 14
GW7, <b>PS4</b>	Mar 21
Essay draft due	Mar 24
GW8, Essay due	Apr 4
GW9, <b>PS5</b>	Apr 11
GW10	Apr 18
<b>PS6</b>	May 2
<b>Final Exam</b>	TBA