

## MATH 23B-1: INTRODUCTION TO PROOFS SPRING 2017

**Instructor:** Arunima Ray (please call me Aru)

**Email:** aruray@brandeis.edu

**Course website:** Piazza

**Class meetings:** MWTh 1p–150p (Block F), Goldsmith 300

**Office:** Goldsmith 205

**Office hours:** M 11a-12noon, W 5-6p, and by appointment

**Graders:** TBA

**Course Assistant:** TBA

**Prerequisites:** Math 15a, 20a, or 22a, or permission of the instructor. Please come see me if you have any concerns about whether you have the right mathematical experience for this course.

**Textbook:** *The art of proof*, Matthias Beck and Ross Geoghegan, Springer, ISBN: 978-1-4419-7022-0.

The book is available online at <http://math.sfsu.edu/beck/papers/aop.noprint.pdf>, as well as on our Perusall site. You may download a PDF from either site, but you should not be able to print it out.

**Exemption exam:** A few students each year have strong enough mathematical backgrounds that they do not need to take Math 23b. We offer the Math 23b exemption exam for them. For more information, see <http://people.brandeis.edu/~charney/23bex.html>

If you would like to attempt the exemption exam, please get in touch with me immediately.

**Purpose:** The foundations of education consists of three R's – Reading, wRiting, and aRithmetic. This course will address all three. We will spend a lot of time Reading our textbook, and wRiting clear and logical arguments; along the way, we will learn some interesting aRithmetic, or rather mathematics, of the sort that all mathematicians should know.

This course is a bridge between your introductory mathematics courses, which are based primarily on computations, and advanced mathematics courses, which are based primarily on proofs. Many people refer to mathematics as a language – in this class you will begin to learn to speak this language. More like an advanced language class, merely memorizing the vocabulary will not suffice (in fact, hopefully we can keep vocabulary to a minimum), but rather you will be required to understand and speak clearly in this language. The material learned here will help you understand the mathematics you read and clarify the mathematics you write. Because we are learning how to write mathematics, exposition will also be a component in your evaluation.

The overarching goal of this course is for you to get a sense of what a professional mathematician does. I am not suggesting that everyone in the class will choose to become a

professional mathematician (although I hope everyone will consider this path), but rather that if you decide to not follow this route, it will be based on an accurate understanding of what that means.

**Learning goals:** In decreasing order of generality, through this course, students will:

- (1) develop a sense of **what it is that professional mathematicians do**,
- (2) become comfortable in **reading mathematics independently**, including understanding abstract definitions and using supporting examples and counterexamples, as well as critically analyzing mathematical arguments with respect to logic and correctness,
- (3) practice **talking about mathematical arguments** with peers and instructors, including presenting their own ideas as well as critiquing others' ideas,
- (4) become comfortable **writing mathematics**, including technical proof-writing as well as expository writing, while meeting rigorous standards of content, organization, coherence, argument and support, and style and mechanics,
- (5) understand and appreciate the **axiomatic method**,
- (6) recognize **different types of proofs**, including direct and indirect proofs, as well as proofs by induction,
- (7) perform **set operations** on finite and infinite collections of sets and be familiar with properties of set operations,
- (8) define and illustrate the concepts of **relations and functions**, particularly injections, surjections, direct and inverse images and inverse functions, equivalence relations and equivalence classes,
- (9) understand the concrete definition of **infinity**, recognize different values of infinity and the relationships between them.

**Grading:** Your grade will be calculated as follows (the scheme which maximizes your personal grade will be used).

Item	MT = Final	Final > MT	Exams > PS
Group/class work	10%	10%	10%
Essay	10%	10%	10%
Participation	10%	10%	10%
Individual problem sets	30%	30%	20%
Midterm	20%	15%	20%
Final exam	20%	25%	30%

If there are any concerns about grading, please see me within one week of getting the assignment/exam back, and before the last day of class (May 3). There might be a curve at the end of the semester based on student performance.

**Course structure:** I am thinking of this course as a lab course. Most of our classwork will be hands-on and in groups. Mondays will usually be lectures while Wednesdays and Thursdays will usually be group work. This will be adjusted if we miss classes for holidays/snow days/etc.

Please bring your textbook to class.

**Group work:** The first set of groups will be created on Jan 23 - this will be our first day of groupwork. There might be some shuffling of groups during the add/drop period. A new set of groups will be created on Mar 13. In class, you will mostly work within your groups on group assignments. These assignments are designed to be completed in class itself. In case you would like extra time to finish your group assignment, you can take them home – group assignments will almost always be due at the beginning of class on Mondays (see schedule). There will be ten group assignments in total.

If you miss any class days, you should clear this with your group ahead of time; together you should determine how you can make up for your absence.

I will be conducting periodic surveys to ensure that each group-member is contributing to their group to the best of their ability; if you are not, you may receive only a fraction of the points your group has earned.

**Individual problem sets:** There will be six individual problem sets (see schedule).

Problem sets will consist primarily of writing proofs. These should be written in complete sentences – exposition will be a large part of your grade. You will have the opportunity to rewrite a few of your solutions to get points back.

You are welcome to work with your groupmates or other classmates on problem sets, but your final writeup must be your own!! In particular, you are not permitted to look up solutions to the problems in any written form, including on the internet. If in doubt about what sources you can use, ask me.

**Late work policy:** Life tends to get in the way of classes sometimes. To help with this, you will have some “late days”. A late day can be applied to submit your work late by at most one day (24 hours) without penalty (I might ask you to submit your work via email if I happen to be out of town or it is a weekend). You are welcome to use multiple late days on the same assignment.

Each student will have **six** late days to be used for the individual problem sets. Each group will have **two** late days for group assignments.

To apply a late day, you must talk to me in person, or send me an email/private message on Piazza clearly stating that you want to use a late day.

If you have chosen not to apply a late day, or have run out of them, I will deduct  $\frac{1}{7}$  of the assignment grade per day it is late. E.g. if you submit a 100-pt assignment two days late and receive a grade of 70, you will receive a score of 50.

**Participation:** Your participation grade will be based on the following. Each item is weighted equally, i.e. correspond to 2% of your final grade.

- (1) In-class activities, including sincerely working with your group, asking/answering questions, participating on Piazza, etc.,
- (2) Reading assignment: there will be a short reading assignment due most Sunday nights at 8p. These will be posted on the course **Perusa11** site. See LATTE for enrollment

instructions. There will be 12 reading assignments. The two lowest scores will be dropped.

- (3) Reflection activities: respond to periodic reflection prompts (graded on completion).
- (4) Essay contest evaluation: participate in the voting for the essay contest (graded on completion).
- (5) Any two of the following list:
  - (a) Visit the Museum of Math in New York City (<http://momath.org/>). (Write a short report, including date of visit, something you saw/learned, some proof of visit, such as a selfie, ticket stub, etc.)
  - (b) Visit the Mathematica exhibit at the Museum of Science in Boston (<http://www.mos.org/exhibits/mathematica>). (Write a short report, including date of visit, something you saw/learned, some proof of visit, such as a selfie, ticket stub, etc.)
  - (c) Attend a math talk (here or elsewhere). I will announce appropriate events as I think/hear about them. (Write a short report, including date, something you saw/learned, etc.)
  - (d) Create or substantially edit a wikipedia page about a math topic/individual. (If editing an article, run it by me to make sure the edits are substantial enough.)
  - (e) Write a publicly available blogpost about a math topic of your choice (this could consist of making your essay publicly available - please don't do this until after the essay contest has ended). For example, you could write about math in a public Facebook post, or on your blog, etc.
  - (f) Give a 5-minute presentation in class related to math/proofs. Please come talk to me if you're interested in doing this. Presentations will occur in March/April.
  - (g) Create something mathematical - this could be a sculpture, or a poem, or a piece of art, or something else that fits this description (if you have ideas about this ask me if it's acceptable). (Write a short report describing your artwork, and show me either the original or a photo of the artwork.)
  - (h) Watch a movie/play, or read a book/paper/etc about mathematics or mathematicians, and write a response paper. The paper should be about a page long and should describe the movie/play, something you learned, something you thought, etc. For a movie/play please include some proof, like selfie, ticket stub, etc.

**Midterm:** The midterm will be in-class during our usual class time on Mar 9. There will be no make-up exams without prior approval and an appropriate reason. The exam will be open-book, open-notes, open-any-documents-you-want-to-bring-with-you, open-brain, closed-friends, closed-enemies, closed-phone, closed-internet.

**Essay:** The essay will be based on a mathematical topic of your choice. The intended audience consists of your classmates - the essay should be related to mathematics but expository in nature, e.g. you might think of it as a blogpost.

The essay should be one-page, typed (margins 1–1.5in, font 10–12pt (figure captions and references 8-12pt)), and can contain pictures and figures. A submission containing any offensive material will be immediately disqualified and you will receive a score of zero for the assignment.

It might be useful to type up your essay in L<sup>A</sup>T<sub>E</sub>X, but you are welcome to use any other typesetting software, such as Word, OpenOffice, GoogleDocs, etc.

You are welcome to talk to anyone you like about your essay (including the Writing Center in the library) as long as it is written in your own words.

The essay will be graded as follows

Submitted essay topic on time	10 points
Submitted first draft on time	10 points
Essay content	40 points
Essay exposition	40 points

We will hold a contest for the essays - voting in this contest is part of your participation grade. (Names will be removed from essays when voting occurs, and you may opt out of the contest). The finalists will receive an extra credit point on their final grade, and the eventual winner will receive an additional prize!

Some possible topics: fractals, equilibrium in economical systems, mathematics in music, a mathematician of your choice, such as Pythagoras, Noether, Galois, Kovalevsky, Wiles, Mirzakhani, etc., growth of populations, the Mobius strip, the Klein bottle, your favorite mathematical puzzle, the Fibonacci sequence, a particular application of a mathematical result, Schrodinger's equation, the book Flatland, imaginary numbers, complex numbers and how they are used in physics, your favorite mathematical proof, etc.

My favorite math blog is <http://blogs.scientificamerican.com/roots-of-unity/>, I recommend starting there, clicking lots of links and googling lots of things.

You are welcome to use a topic from our textbook, but if it is a topic that we discuss in class, you will be graded more strictly on the content.

**Extra credit:** You can earn up to three points on your final grade in the following ways.

- (1) Be a finalist in the essay contest (see above)
- (2) Any of the activities listed under item (5) in the participation list above.
- (3) I'm open to other possibilities - if you think you should get extra credit for something, let me know and there's a good chance I'll agree with you.

The above will be graded primarily on completion – i.e. if you complete the activity in good faith, you will get your points.

**Discussions:** We will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from classmates, the TA, and myself. Rather than emailing questions to me, I encourage you to post your questions on Piazza. Find our class page at: [piazza.com/brandeis/spring2017/math23b1](http://piazza.com/brandeis/spring2017/math23b1)

**Feedback:** If you are at all concerned about anything related to our course (or anything else), please feel free to send me an email, a message on Piazza, or just drop by my office. Please remember, the sooner you tell me your concerns, the more I can do about them.

In addition, you can send me anonymous feedback through the google form available at <http://goo.gl/forms/HUHQaWRDXk> (this link can also be found on LATTE/Piazza).

**Expectations:** Students are expected to complete readings on time, attend class, work sincerely with their groupmates to the best of their ability. Success in this 4 credit hour course is based on the expectation that students will spend a minimum of 9 hours of study time per week in preparation for class (readings, papers, discussion sections, preparation for exams, etc.).

In return, my goal is for each and every one of you to do well in this course, in the sense of learning something new and fun, and also getting a good grade.

**Disability support:** If you are a student with a documented disability on record at Brandeis University and wish to have a reasonable accommodation made for you in this class, please see me immediately.

**Academic integrity:** You are expected to be familiar with, and to follow, the University's policies on academic integrity. Please consult Brandeis University Rights and Responsibilities for all policies and procedures. All policies related to academic integrity apply to in-class and take home projects, assignments, exams, and quizzes. Students may only collaborate on assignments with permission from the instructor. Allegations of alleged academic dishonesty will be forwarded to the Director of Academic Integrity. Sanctions for academic dishonesty can include failing grades and/or suspension from the university.

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**TL;DR:**

- (1) There is a reading assignment due on **Perusall** every Sunday at 10p. Comment on these for participation points
- (2) Mondays will be mostly lecture, Wednesdays and Thursdays will be mostly group work.
- (3) Groups will turn in their class work roughly every week.
- (4) There is an individual problem set due roughly every two weeks.
- (5) There is one in-class midterm in early March.
- (6) There is a one-page paper due in April. Finalists get extra credit, winner gets additional prize. Vote in the contest for participation points.
- (7) Participate in the discussion forum on Piazza to get help on questions and for participation points.
- (8) There are many extra credit opportunities.
- (9) Come talk to me if you have any concerns.

**Course Schedule:**

Groups created	Jan 23
GW1	Jan 30
GW2, <b>PS1</b>	Feb 6
GW3	Feb 13
<b>PS2</b>	Feb 16
GW4	Feb 27
GW5, <b>PS3</b>	Mar 6
<b>Midterm</b>	Mar 9
New groups created	Mar 13
GW6, Essay topics due	Mar 20
GW7, <b>PS4</b>	Mar 27
Essay drafts due	Mar 30
GW8	Apr 3
<b>PS5</b>	Apr 6
Essays due, GW9	Apr 19
<b>PS6</b>	Apr 24
GW10	May 1
<b>Final Exam</b>	TBA